Sample Question Paper - 25
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

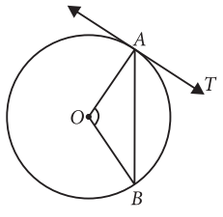
1. If the price of a book is reduced by ₹ 5, a person can buy 5 more books for ₹ 300. Find the original marked price of the book.

2. In a continuous frequency distribution, the median of the data is 21. If each observation is increased by 5, then find the new median.

3. Two parallel lines touch the circle at points A and B. If area of the circle is 16π cm^2, then find the length of AB.

OR

In the given figure, O is the centre of the circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100°$, then find $\angle BAT$.



4. There are three sections A, B and C in class X with 25, 40 and 35 students respectively. The mean marks obtained by section A, B and C are 70%, 65% and 50% respectively. Find the mean marks of entire class X.

5. A cone and a sphere have equal radii and equal volume. What is the ratio of the diameter of the sphere to the height of cone?

6. If $\frac{2}{3}$, k, $\frac{5k}{8}$ are in A.P., then find the value of k.

OR

How many terms of the A.P. : 9,17,25, ... must be taken to give a sum 636?

SECTION - B

7. Solve for x : $3^{2x+3} - 730(3^x - 1) = 703$.

8. The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 m, then find the height of the chimney.

OR

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.

9. Draw a circle of radius 8 cm and draw a tangent to this circle making an angle of 55° with a line passing through the centre.

10. The sum of first 20 terms of an A.P. is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

SECTION - C

11. The rainwater from a roof of 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full, find the height of the rainfall in cm.

OR

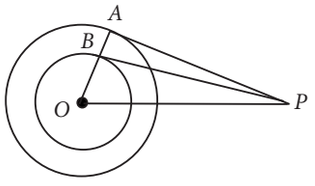
A solid consists of a circular cylinder with an exact fitting right circular cone placed at the top. The height of the cone is h. If the total volume of the solid is 3 times the volume of the cone, then what is the height of the circular cylinder?

12. An aeroplane flying at a height of 9000 m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two planes from the same point on the ground are 60° and 30° respectively. Find the vertical distance between the aeroplanes at that instant.

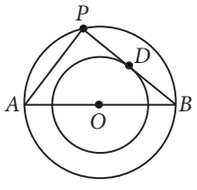
Case Study - 1

13. If a tangent is drawn to a circle from an external point, then the radius at the point of contact is perpendicular to the tangent. Answer the following questions using the above condition.

(i) In the given figure, O is the centre of two concentric circles. From an external point P tangents PA and PB are drawn to these circles such that $PA = 6$ cm and $PB = 8$ cm. If $OP = 10$ cm, then find AB.



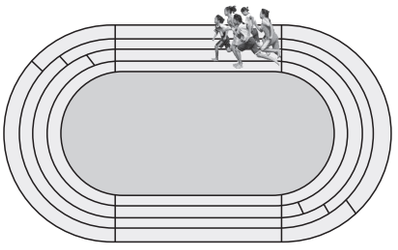
(ii) The diameter of two concentric circles are 10 cm and 6 cm. AB is a diameter of the bigger circle and BD is the tangent to the smaller circle touching it at D and intersecting the larger circle at P on producing. Find the length of BP.



Case Study - 2

14. Coach wants to analyses the time taken by a student to complete the race track of 100 m, for which he recorded the data of 50 students, which is given below.

Time (in minutes)	14.5-19.5	19.5-24.5	24.5-29.5	29.5-34.5	34.5-39.5
No. of students	10	15	12	8	5



Based on the above information, answer the following questions :

(i) Find the mean time taken by the students to complete the track.

(ii) Find the modal class of the given data.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. Let the marked price of the book be ₹ x.

Total cost = ₹ 300

$\therefore$ Number of books = $\frac{300}{x}$

If price of the book is ₹ $(x - 5)$, then

Number of books $= \frac{300}{x-5}$

According to question,

$$\frac{300}{x-5} - \frac{300}{x} = 5 \Rightarrow \frac{300x - 300(x-5)}{(x-5)x} = 5$$

$\Rightarrow 1500 = 5(x^2 - 5x) \Rightarrow x^2 - 5x - 300 = 0$

$\Rightarrow x^2 - 20x + 15x - 300 = 0$

$\Rightarrow x(x - 20) + 15(x - 20) = 0$

$\Rightarrow (x - 20)(x + 15) = 0 \Rightarrow x = 20$ or $x = -15$

Since, x has to be a positive integer, so $x = -15$ is rejected.

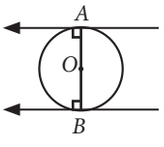
$\therefore x = 20$

Hence, original marked price of the book is ₹ 20.

2. If each observation of a continuous frequency distribution is increased by a number, then median is also increased by the same number.

$\therefore$ New median = 21 + 5 = 26

3. Let the radius of the circle be r cm.



Area of circle = 16π cm^2 [Given]

$\Rightarrow \pi r^2 = 16\pi \Rightarrow r^2 = 16 \Rightarrow r = 4$

$\therefore AB = 2\,OA = 2r = 8$ cm

OR

Given, $\angle AOB = 100°$

Now, $OA = OB \Rightarrow \angle OAB = \angle OBA$ ... (i)

In ΔAOB, $\angle AOB + \angle OAB + \angle OBA = 180°$

$\Rightarrow 100° + \angle OAB + \angle OAB = 180°$ [Using (i)]

$\Rightarrow 2\angle OAB = 80° \Rightarrow \angle OAB = 40°$

Now, $\angle OAT = 90°$ [$\because$ Tangent at any point of a circle is perpendicular to the radius through point of contact]

Thus, $\angle BAT = \angle OAT - \angle OAB = 90° - 40° = 50°$

4. Mean marks obtained by section A = 70%

$\therefore$ Sum of marks of 25 students = 70% × 25 = 1750%

Mean marks obtained by section B = 65%

$\therefore$ Sum of marks of 40 students = 40 × 65% = 2600%

Mean marks obtained by section C = 50%

$\therefore$ Sum of marks of 35 students = 35 × 50% = 1750%

Mean marks of class X

$$= \frac{\text{Sum of marks of section } A, B, C}{\text{Total number of students}}$$

$$= \frac{(1750 + 2600 + 1750)\%}{25 + 40 + 35} = \left(\frac{6100}{100}\right)\% = 61\%$$

5. Let r be the radius of sphere and cone and h be the height of the cone.

$\because$ Volume of sphere = Volume of cone [Given]

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h \Rightarrow 4r = h$

$\therefore$ Height of cone = $4r$

Also, diameter of sphere = $2r$

$\therefore$ Required ratio $= \frac{2r}{4r} = \frac{1}{2} = 1 : 2$

6. Given, $\frac{2}{3}$, k, $\frac{5k}{8}$ are in A.P.

$\Rightarrow k - \frac{2}{3} = \frac{5k}{8} - k \Rightarrow k + k - \frac{5k}{8} = \frac{2}{3}$

$\Rightarrow \frac{11k}{8} = \frac{2}{3} \Rightarrow k = \frac{16}{33}$

OR

We have, A.P. 9, 17, 25, ...

And $S_n = 636$

Here, $a = 9$ and $d = a_2 - a_1 = 17 - 9 = 8$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 636 = \frac{n}{2}[2 \times 9 + (n-1)8]$

$\Rightarrow 1272 = n[18 + 8n - 8]$

$\Rightarrow 1272 = n[10 + 8n]$

$\Rightarrow 8n^2 + 10n - 1272 = 0$

$\Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow (n - 12)(4n + 53) = 0$

$\Rightarrow n = 12$ or $-\frac{53}{4}$ (Not possible)

$\therefore n = 12$

7. We have, $3^{2x+3} - 730(3^x - 1) = 703$

$\Rightarrow 27(3^x)^2 - 730(3^x) + 730 - 703 = 0$

$\Rightarrow 27t^2 - 730t + 27 = 0$ [Putting $3^x = t$]

$\Rightarrow 27t^2 - 729t - t + 27 = 0$

$\Rightarrow 27t(t - 27) - 1(t - 27) = 0$

$\Rightarrow \quad (t-27)(27t-1)=0 \quad \Rightarrow \quad (t-27)(27t-1)=0$

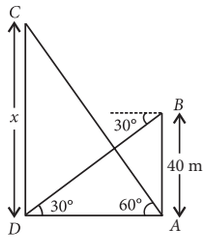
$\Rightarrow \quad t=27 \text{ or } t=\dfrac{1}{27} \quad \therefore \; 3^x=27 \text{ or } 3^x=\dfrac{1}{27}$

$\Rightarrow \quad 3^x=3^3$

or $\quad 3^x=3^{-3} \Rightarrow x=3 \text{ or } x=-3$

8. Let AB be the tower and $CD = x\ m$ be the height of the chimney.

In ΔABD, we have



$$\tan 30^\circ = \frac{AB}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{40}{AD}$$

$$\Rightarrow \quad AD = 40\sqrt{3} \text{ m} \qquad \text{...(i)}$$

$$\text{In } \Delta CDA, \tan 60^\circ = \frac{CD}{AD}$$

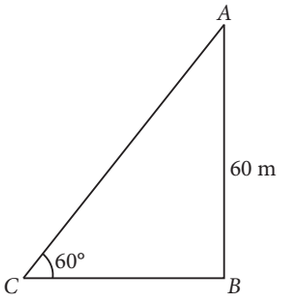
$$\Rightarrow \quad \sqrt{3} = \frac{x}{40\sqrt{3}} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad x = 40 \times 3 = 120$$

$\therefore$ Height of chimney = 120 m

OR

Let A be the position of kite and CA be the string attached to the kite such that its one end is tied to a point C on the ground. The inclination of the string CA with the ground is 60°.



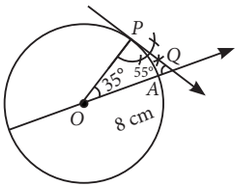
In ΔABC, we have

$$\sin C = \frac{AB}{AC} \Rightarrow \sin 60^\circ = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

$$\Rightarrow \quad AC = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is $40\sqrt{3}$ m.

9. Steps of construction :



Step-I : Draw a circle with centre O and radius 8 cm.

Step-II : Draw a radius OA.

Step-III : Draw an $\angle AOP$ equal to the complement of 55° *i.e.*, equal to 35°.

Step-IV : Draw a perpendicular OP at P which intersects OA produced at point Q.

Hence, PQ is the required tangent such that $\angle OQP = 55^\circ$.

10. Let the first term of the A.P. be a and common difference be d.

$\therefore$ The sum of first 20 terms is,

$$S_{20} = \frac{20}{2}[2a + (20-1)d]$$

$$\Rightarrow \quad 400 = 10(2a + 19d) \Rightarrow \; 2a + 19d = 40 \qquad \text{...(i)}$$

$$\text{Also, } S_{40} = \frac{40}{2}(2a + 39d)$$

$$\Rightarrow \quad 1600 = 20(2a + 39d)$$

$$\Rightarrow \quad 2a + 39d = 80 \qquad \text{...(ii)}$$

On solving (i) and (ii), we get

$a = 1$ and $d = 2$

$$\therefore S_{10} = \frac{10}{2}[2\times 1 + (10-1)(2)]$$

$$= 5\,(2 + 9 \times 2) = 5\,(2 + 18) = 5 \times 20 = 100$$

11. Length of roof = 22 m, breadth of roof = 20 m

Let the height of the rainfall be x cm.

$$\text{Volume of water on the roof} = \left(22 \times 20 \times \frac{x}{100}\right)\text{m}^3$$

$$= \frac{22x}{5}\text{m}^3$$

Radius of the base of the cylindrical vessel = 1 m

Height of the cylindrical vessel = 3.5 m

Volume of water in the cylindrical vessel when it is just full $= \left(\dfrac{22}{7} \times 1 \times 1 \times \dfrac{7}{2}\right)\text{m}^3 = 11\ \text{m}^3 \quad [\because\ V = \pi r^2 h]$

Now, volume of water on the roof = volume of water in the vessel

$$\Rightarrow \quad \frac{22x}{5} = 11 \quad \Rightarrow \quad x = \left(\frac{11 \times 5}{22}\right) = 2.5$$

Hence, height of the rainfall is 2.5 cm.

OR

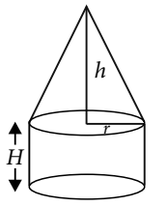
Let H be the height of cylinder

Since, volume of solid = 3 × Volume of cone

$= 3 \times \frac{1}{3}\pi r^2 h = \pi r^2 h$

Now, volume of the cylinder

= Volume of solid – Volume of the cone



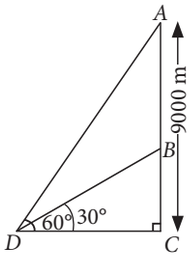
$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

$\Rightarrow \pi r^2 H = \frac{2}{3}\pi r^2 h \Rightarrow H = \frac{2}{3}h$

Hence, height of the cylinder $= \frac{2h}{3}$

12. Let A and B be the positions of two aeroplanes when A is vertically above B and $AC = 9000$ m.

Let D be the point of observation on the ground such that $\angle ADC = 60°$ and $\angle BDC = 30°$.



In ΔACD, $\tan 60° = \frac{AC}{CD}$

$\Rightarrow \sqrt{3} = \frac{9000}{CD}$

$\Rightarrow CD = \frac{9000}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3000\sqrt{3}$ m …(i)

In ΔBCD, $\tan 30° = \frac{BC}{CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{3000\sqrt{3}}$ [From (i)]

$\Rightarrow BC = 3000$ m

$\therefore$ Vertical distance between A and $B = AB = AC - BC = 9000 - 3000 = 6000$ m

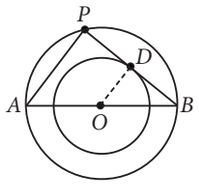
13. (i) Here, $OP^2 - PB^2 = OB^2$ and $OP^2 - PA^2 = OA^2$

$\therefore \quad OB = \sqrt{100 - 64} = \sqrt{36} = 6$ cm

and $OA = \sqrt{100 - 36} = \sqrt{64} = 8$ cm

$\therefore \quad AB = OA - OB = 8 - 6 = 2$ cm

(ii) Join OD



Here, in right angled ΔOBD, $OB = 5$ cm and $OD = 3$ cm.

$\therefore \quad BD = \sqrt{25 - 9} = \sqrt{16} = 4$ cm

Since, chord BP is bisected by radius OD.

$\therefore \quad BP = 2BD = 8$ cm

14.

Class interval	Class mark (x_i)	Frequency (f_i)	$f_i x_i$
14.5-19.5	17	10	170
19.5-24.5	22	15	330
24.5-29.5	27	12	324
29.5-34.5	32	8	256
34.5-39.5	37	5	185
	Total	50	1265

(i) Required mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1265}{50} = 25.3$ min.

(ii) Here maximum frequency is 15, which lies in the interval 19.5-24.5.

$\therefore$ Modal class is 19.5-24.5.